Aspects of PDE-constrained multi-disciplinary optimization

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Two complementary numerical approaches are proposed for the treatment of multi-objective optimization problems originating from PDE-constrained multi-disciplinary design optimization, assuming the same set of N design variables is shared by the n different physics ($n \geq 1$; $N \geq 1$; n and N independent).

Nash games were introduced in [8] as a formulation to achieve a physically-relevant trade-off between concurrent objectives, mostly in the context of aerodynamic shape optimization, when the different objectives can be associated with localized phenomena. In [2] and [5], a *competitive strategy* has been developed to rationalize this approach and make it more robust, in the context of two-discipline optimization problems in which one discipline, the primary discipline, is preponderant, or fragile. It was then recommended to identify, in a first step, the optimum of this discipline alone using the whole set of design variables. Then, an orthogonal basis is constructed based on the evaluation at convergence of the Hessian matrix of the primary criterion and constraint gradients. This basis is used to split the working design space into two supplementary subspaces to be assigned, in a second step, to two virtual players in competition in an adapted Nash game, devised to reduce a secondary criterion while causing the least degradation to the first. The formulation has been proved to potentially provide a set of Nash equilibrium solutions originating from the original single-discipline optimum point by smooth continuation, thus introducing competition gradually. This method has been illustrated in shape optimization problems involving steady compressible aerodynamics (drag minimization under lift constraint) versus either structural design [1], and more recently sonic-boom reduction, or unsteady aerodynamics.

As a complement to the above method, a *cooperative algorithm*, the Multiple-Gradient Descent Algorithm (MGDA) was originally proposed in [3] for the treatment of multi-objective *differentiable optimization*. It was tested and reformulated in [7]. Given the gradients of n objective functions, a simple result of convex analysis permits us to identify a vector ω in the direction of which the Fréchet derivatives of the n objective functions are all positive. MGDA uses $-\omega$ as a natural descent direction. The capability of the basic method to identify Pareto-optimal solutions has been established theoretically and numerically in [9]. A number of enhancements of the basic method have since been proposed. In *MGDA-II*, the descent direction is calculated by a direct procedure [6] based on a Gram-Schmidt orthogonalization process (*GSP*) with special normalization. This algorithm was tested in the context of a simulation by domain partitioning, as a technique to match the different interface components concurrently [4]. Two other variants have also been proposed. The first, *MGDA-III*, realizes two enhancements. Firstly, the *GSP* is conducted incompletely whenever a test reveals that the current estimate of the direction of search is adequate also w.r.t. the gradients not yet

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taken into account; this improvement simplifies the identification of the search direction when the gradients point roughly in the same direction, and makes the Fréchet derivative common to several objective-functions larger. Secondly, the order in which the different gradients are considered in the *GSP* is defined in a unique way devised to favor an incomplete *GSP*. In the second variant, *MGDA-IV*, the question of scaling is addressed when the Hessians are known. In this context, the optimal step-size has also been identified. A variant is also proposed in which the Hessians are estimated by the Broyden-Fletcher-Goldfarb-Shanno (*BFGS*) formula. Lastly, we consider the convergence of the method when equality constraints are treated by penalization.

The potential of combining the two methods to identify a large portion of the Pareto front will be demonstrated in the case of the optimum-shape design of a supersonic configuration with respect to the concurrent minimization of drag (under lift constraint) calculated by the finite-volume simulation of 3D compressible Euler equations and a sonic-boom criterion calculated by wave propagation from the near-field pressure distribution.

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